# 12

# Predicting

## Chapter Overview

Engineers have always needed to be able to predict the behaviours of their creations, making the ability to predict system behaviour an integral part of an engineer's tool box. This prediction can be accomplished in many ways. In the past, engineers often extrapolated from their past experience to predict how their designs and solutions will perform in different conditions. Some branches of engineering, such as aerodynamics, make heavy use of physical models to support their design and decision-making. The most common prediction approach is mathematical modelling, which is the focus of this chapter.

In mathematical modelling, a physical system is represented by one or more equations. When solved, these equations predict the behaviour of the physical system in different situations. Sometimes, you will use equations from textbooks and references directly. At other times, you will need to modify the equations, or create mathematical models yourself. Developing an understanding of the modelling process can aid you in the aforementioned endeavours.

The process of developing a mathematical model is common across all disciplines and consists of six steps: [1] subdivide the physical system into spatial components and temporal phases, [2] make assumptions to simplify the model, [3] identify relationships, [4] identify the variables associated with these relationships, [5] validate the model, and [6] solve the model to yield predictions. Of the six steps, the first two steps require the most thought and experience to do well.

The goal of making assumptions is to intentionally ignore some physical phenomena to simplify models. This is the most critical part of modelling, and both knowledge and experience are needed to do it well. The price of making assumptions is the loss of prediction accuracy. An over-simplified model is unacceptable because the inaccuracy introduced can mislead the decisions that relies on the predictions of the model.

The basis of mathematical modelling is that the physical world is highly predictable, such that we can predict the behaviours of many diverse systems using a relatively small number of rules and laws. This is a belief we derive from centuries of experience. Relationships, or equations, are our expressions of these rules and laws using the language of mathematics; these relationships, when applied to a problem, constrain the solution space of the problem. When sufficiently constrained, the problem yields a single solution – the prediction. Engineers work with many different kinds of relationships, including conservation laws, state equations, heuristic relationship, compatibility requirements, and derived relationships.

Conservation laws represent the knowledge that certain quantities, such as mass, energy, momentum, or charge, cannot be created or destroyed. These laws hold true in all situations; when developing models, we only need to know how to apply them and if they are useful. We should also note two special cases when modelling a component: a component is closed with respect to a certain quantity if no flow of this quantity enters or leaves the component, and a component is in steady state when its properties do not change with time. A state equation describes the relationship between two characteristics of a physical object. A heuristic is a relationship developed based on many years of observation rather than fundamental principles, and may not be expressed as equations. A compatibility relationship is used to maintain compatibility across multiple subdivisions of the overall model. Derived relationships are often-used relationships and equations that are derived from first principles. Occasionally, there are also relationships that do not fit into this categorization system.

Whereas equations represent the relationships between physical characteristics of the modelled systems, variables are the quantitative representations of these characteristics. Exogenous variables encompass all quantities that are external to a problem, and are known prior to the modelling process. Parameters are a special kind of exogenous variable that moves a model along a continuous path, often time. The purpose of the modelling process is to find values for endogenous variables of that all relationships are satisfied; when the number of relationships is higher than the number of endogenous variables, the difference between design variables the engineer can choose freely to achieve design objectives.

Often, an engineering problem is broken down to spatial components or temporal phases, modeled individually, and reassembled afterward into a complete model. Maintaining compatibility between components and phases is necessary for the reassembly, and can often be accomplished by using the same variables across components and phases. When three or more components or phases share a common boundary, compatibility relationships are used.

Before applying the model for decision-making purposes, it should be validated. The quickest check of a model is to ensure unit consistency. Using sensitivity analysis, we can ensure the models behave reasonably in response to changes in variables. Another way to validate the model is to compare its predictions with physical models, real world experience, or predictions of similar models. Lastly, once the model is solved, we should make sure that the determined endogenous variables have magnitudes that are reasonable based on our experience.

There are many ways to solve mathematical models. The textbook introduces a number of algebraic approaches for manipulating equations for making solving them easier. Today, most mathematical models used in engineering are solved using computer programs running pre-existing algorithms called "solvers."

## Learning Objectives

In this chapter, you will:

- learn about the role of prediction in engineering;
- explore mathematical modeling as a process;
- learn about different types of mathematical relationships in modeling, including conservation laws, state equations, heuristic relationship, compatibility requirements, and derived relationships; and
- learn about distinguish exogenous, endogenous, and parametric variables.