



APPENDIX

Key Formulas

Learn to
use this
formula on:

ANOVA (One-way Analysis of Variance)

Total sum of squares (version 1)	$SS_{total} = SS_{between\ group} + SS_{within\ group}$	p. 234
Total sum of squares (version 2)	$SS_{total} = \sum(x_i - \bar{x}_{total})^2$	p. 238
Between group sum of squares	$SS_{between\ group} = \sum n_{group} (\bar{x}_{group} - \bar{x}_{total})^2$	p. 237
Between group degrees of freedom	$df_{between\ group} = (\text{number of groups} - 1)$	p. 239
Within group sum of squares	$SS_{within\ group} = \sum(x_i - \bar{x}_{group})^2$	p. 235
Within group degrees of freedom	$df_{within\ group} = (n_{group1} - 1) + (n_{group2} - 1) + (n_{group3} - 1) + \dots$	p. 239
F-statistic (or F-ratio)	$F = \frac{SS_{between\ group} / df_{between\ group}}{SS_{within\ group} / df_{within\ group}}$	p. 240

Chi-square test of independence

Chi-square statistic	$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$	p. 280
Degrees of freedom of the chi-square statistic	$df_{\chi^2} = (\text{rows} - 1)(\text{columns} - 1)$	p. 287
Expected frequencies	$f_e = \frac{(\text{row total})(\text{column total})}{\text{overall total}}$	p. 280

Cohen's d

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s} \quad \text{p. 201}$$

Correlation coefficient, Pearson's

Pearson correlation coefficient (version 1)	$r = \frac{SP}{\sqrt{SS_x} \sqrt{SS_y}}$	p. 320
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Pearson correlation coefficient (version 2)	$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2} \sqrt{\sum(y_i - \bar{y})^2}}$	p. 320
Sum of products	$SP = \sum(x_i - \bar{x})(y_i - \bar{y})$	p. 318
Sum of squares (x)	$SS_x = \sum(x_i - \bar{x})^2$	p. 319
Sum of squares (y)	$SS_y = \sum(y_i - \bar{y})^2$	
T-statistic for Pearson's correlation coefficient	$t = r \sqrt{\frac{n-2}{1-r^2}}$	p. 323
Degrees of freedom of the t-statistic for Pearson's correlation coefficient	$df_t = n - 2$	p. 327

Correlation coefficient, Spearman's rank-order

Spearman's rank-order correlation coefficient (version 1)	$\rho = \frac{SP}{\sqrt{SS_x} \sqrt{SS_y}}$	p. 330
	<i>Note: Ranks are used in place of values and means in the calculation of the sum of products and sum of squares.</i>	
Spearman's rank-order correlation coefficient (version 2)	$\rho = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2} \sqrt{\sum(y_i - \bar{y})^2}}$	p. 330
	<i>Note: Ranks are used in place of values and means.</i>	
Sum of products	$SP = \sum(x_i - \bar{x})(y_i - \bar{y})$	p. 318
	<i>Note: Ranks are used in place of values and means.</i>	
Sum of squares (x)	$SS_x = \sum(x_i - \bar{x})^2$	p. 319
	<i>Note: Ranks are used in place of values and means.</i>	
Sum of squares (y)	$SS_y = \sum(y_i - \bar{y})^2$	p. 319
	<i>Note: Ranks are used in place of values and means.</i>	
T-statistic for Spearman's rank-order correlation coefficient	$t = \rho \sqrt{\frac{n-2}{1-\rho^2}}$	p. 332
Degrees of freedom of the t-statistic for Spearman's rank-order correlation coefficient	$df_t = n - 2$	p. 335

Covariance

$$\text{cov}(x, y) = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n-1} \quad \text{p. 318}$$

Cramér's V

$$V = \sqrt{\frac{\chi^2}{n(\min[\text{rows}-1], [\text{columns}-1])}} \quad \text{p. 289}$$

Gamma

$$\gamma = \frac{\text{Concordant pairs} - \text{Discordant pairs}}{\text{Concordant pairs} + \text{Discordant pairs}} \quad \text{p. 272}$$

Lambda

$$\lambda = \frac{E_1 - E_2}{E_1} \quad \text{p. 266}$$

$$E_1 = n_{\text{overall}} - \text{modal frequency}_{\text{overall}}$$

$$E_2 = \sum(n_{\text{group}} - \text{modal frequency}_{\text{group}})$$

Means (averages)

Mean (average)

$$\bar{X} = \frac{\sum X_i}{N} \quad \text{or} \quad \bar{x} = \frac{\sum x_i}{n} \quad \text{p. 98}$$

Standard error of a mean

$$se_{\bar{x}} = \frac{s}{\sqrt{n}} \quad \text{p. 163}$$

95% confidence interval for a mean

$$95\% CI_{\bar{x}} = \bar{x} \pm 1.96(se_{\bar{x}}) \quad \text{p. 168}$$

Percentage

$$\% = \left(\frac{F}{N}\right)100 \quad \text{p. 31}$$

Phi

$$\phi = \sqrt{\frac{\chi^2}{n}} \quad \text{p. 288}$$

Probability

Simple probability

$$\text{probability (outcome of interest)} = \frac{\text{number of outcomes of interest}}{\text{number of possible outcomes}} \quad \text{p. 132}$$

Joint probability

$$\begin{aligned} &\text{probability (outcome of interest, outcome of interest)} \\ &= \frac{\text{number of outcomes of interest}}{\text{number of possible outcomes}} \times \frac{\text{number of outcomes of interest}}{\text{number of possible outcomes}} \quad \text{p. 134} \end{aligned}$$

Proportions

Proportion

$$p = \left(\frac{F}{N}\right) \quad \text{or} \quad p = \left(\frac{f}{n}\right) \quad \text{p. 31}$$

Standard error of a proportion $se_p = \sqrt{\frac{p(1-p)}{n}}$ p. 182

95% confidence interval for a proportion $95\% CI_p = p \pm 1.96(se_p)$ p. 185

R² (Coefficient of determination)

$R^2 = \frac{\sum(\hat{y}_i - \bar{y})^2}{\sum(y_i - \bar{y})^2}$ p. 367

Adjusted $R^2 = 1 - \left[\frac{(1 - R^2)(n - 1)}{n - k - 1} \right]$ p. 455

Rates

Rate (common base) $\frac{F}{N} = \frac{x}{\text{base of the rate}}$ p. 37

Rate (standardized numerator) $\frac{F}{N} = \frac{1}{x}$ p. 37

Ratio

$\text{ratio} = \frac{F_1}{F_2}$ p. 36

Regression, Simple linear

Simple linear regression predictions $\hat{y} = a + bx$ p. 356

Simple linear regression with residuals $y_i = a + bx_i + e_i$ p. 365

Standard error of the estimate $se_{est} = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n - 2}}$ p. 372

Sum of products $SP = \sum(x_i - \bar{x})(y_i - \bar{y})$ p. 318

Sum of squares (x) $SS = \sum(x_i - \bar{x})^2$ p. 319

Constant coefficient $a = \bar{y} - b(\bar{x})$ p. 363

Standard error of the constant coefficient (version 1) $se_a = se_{est} \sqrt{\frac{1}{n} + \frac{(\bar{x})^2}{SS_x}}$ p. 373

Standard error of the constant coefficient (version 2) $se_a = se_{est} \sqrt{\frac{1}{n} + \frac{(\bar{x})^2}{\sum(x_i - \bar{x})^2}}$ p. 373

Slope coefficient (version 1) $b = \frac{SP}{SS_x}$ p. 360

Slope coefficient (version 2)	$b = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$	p. 360
Standard error of a slope coefficient (version 1)	$se_b = \frac{se_{est}}{\sqrt{SS_x}}$	p. 373
Standard error of a slope coefficient (version 2)	$se_b = \frac{se_{est}}{\sqrt{\sum(x_i - \bar{x})^2}}$	p. 373
95% confidence intervals for regression coefficients (large samples)	$95\%CI_b = b \pm 1.96(se_b)$ $95\%CI_a = b \pm 1.96(se_a)$	p. 378
T-statistic for regression coefficients	$t = \frac{b}{se_b} \quad \text{or} \quad t = \frac{a}{se_a}$	p. 374
Degrees of freedom of the t-statistic for regression coefficients	$df_t = n - k - 1$	p. 377

Regression, multiple linear

Multiple linear regression predictions	$\hat{y} = a + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + \dots$	p. 407
Multiple linear regression with residuals	$y_i = a + b_1x_{1i} + b_2x_{2i} + b_3x_{3i} + b_4x_{4i} + b_5x_{5i} + \dots + e_i$	p. 444
Constant coefficient (two independent variables)	$a = \bar{y} - b_1\bar{x}_1 - b_2\bar{x}_2$	p. 412
Partial slope coefficients (two independent variables)	$b_{x_1} = \left(\frac{r_{x_1,y} - [r_{x_1,x_2}][r_{x_2,y}]}{1 - [r_{x_1,x_2}]^2} \right) \left(\frac{s_y}{s_{x_1}} \right)$ $b_{x_2} = \left(\frac{r_{x_2,y} - [r_{x_1,x_2}][r_{x_1,y}]}{1 - [r_{x_1,x_2}]^2} \right) \left(\frac{s_y}{s_{x_2}} \right)$	p. 410
Standardized slope coefficient (Beta)	$\beta_x = b_x \left(\frac{s_x}{s_y} \right)$	p. 416

Standard deviation

Standard deviation (using sample data)	$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n - 1}}$	p. 140
Standard deviation (using population data)	$S = \sqrt{\frac{\sum(X_i - \bar{X})^2}{N}}$	p. 102

Standardized weight

$$\text{standardized weight} = \frac{\text{weight for each case}}{\text{mean of the weight variable}} \quad \text{p. 148}$$

T-statistic (unequal variances)

T-statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{p. 210}$$

Degrees of freedom of the t-statistic

$$df_t = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{(n_1-1)} + \frac{(s_2^2/n_2)^2}{(n_2-1)}} \quad \text{p. 215}$$

Variance

Variance (using sample data)

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} \quad \text{p. 140}$$

Variance (using population data)

$$S^2 = \frac{\sum(X_i - \bar{X})^2}{N} \quad \text{p. 103}$$

Z-score

$$Z = \frac{(X_i - \bar{X})}{S} \quad \text{p. 105}$$