

# Supplementary Section 6S.7

## The Propositions of Propositional Logic

The central concern in *Introduction to Formal Logic with Philosophical Applications* is logical consequence: What follows from what? Relatedly, if not quite equivalently, we are seeking the theorems, or laws, of logic. But of what do the laws of logic hold? The variables in **PL** are propositional variables; they stand for propositions. The laws of logic are complex propositions, and the entailments we prove are among propositions too. But what are propositions?

In this section, we'll look briefly at some views about the controversial question of the nature of propositions in order to understand better the subject matter of logic. We'll find that propositions seem to have an odd status, not concrete sentences or mental objects, but between the two.

### LOGIC AND THE LAWS OF THOUGHT

Before Frege, it was common to think of logic as the laws of thought. The claim that logic summarizes rules of thought is ambiguous between a descriptive interpretation and a prescriptive one. On the descriptive sense, logic is supposed to describe how we actually reason. On this view, we could find the laws of logic by surveying opinions about correct inferences. Unfortunately, people's capacities for reasoning are limited. We make logical errors. On the descriptive interpretation of logic, if enough people make an error, we might be forced to see it as a rule of logic.

On a prescriptive interpretation of taking logic to be the rules of thought, logic tells us what thoughts should follow from which others, or which thoughts should be true just because of their structure. This claim is perhaps more plausible, but still not right. Most of the infinitely many logical truths are too complex to think. Moreover, in many cases, we do not choose what to think. Our thoughts and their sequence are influenced by environment and chemistry in ways that derivations in logic are impotent to redirect.

Moreover, if the laws of logic hold of ideas, then they are essentially subjective, whether we hold this claim to be descriptive or prescriptive. As Frege argued, any view that makes the laws of logic hold of thoughts makes logic subservient to psychology.

But logic, as we use it, is more basic or fundamental than psychology. It holds for all disciplines.

Many more-recent philosophers, including Frege, believe that the laws of logic do not concern relations among thoughts in our minds, but the objective nature of entailments. Whether propositional variables stand for psychological claims, or physical ones, or religious ones, or whatever, logic tells us which complex propositions are true just because of their structure and which entailments hold just because of the relations among the simple components used in the inferences. They hold for any claims, for any people.

## LAWS OF LOGIC AND THEIR INSTANCES

To say that the laws of logic are objective, though, does not determine what they hold of. Consider a typical simple theorem of logic, the law of the excluded middle, at 6S.7.1.

$$6S.7.1 \quad \alpha \vee \sim\alpha$$

6S.7.1 is a sentence in our logical metalanguage. It can be thought of as a schema, a rule for generating particular theorems of our object language: ‘ $P \vee \sim P$ ’; ‘ $Pa \vee \sim Pa$ ’; ‘ $(\exists x)Px \vee \sim(\exists x)Px$ ’. Laws like 6S.7.1, expressed as schemas written in the metalanguage, tell you how to form certain propositions, or how to infer some propositions from others. We write the law of the excluded middle as 6S.7.1 as a way of saying that any substitution instance of that metalinguistic sentence is a tautology.

The substitution instances we form from logical laws are sometimes called logical truths. (Sometimes we call the incomplete sentence of the metalanguage a logical truth as well.) Many sentences, like ‘It is raining now’, require for their truth some justification outside of logic. It must actually be raining in order for that sentence to be true. Logical truths, in contrast, require no justification outside of logic. They can be shown true using our bivalent semantics. And they can be derived regardless of any assumptions or premises. In fact, they can be derived with no premises at all using the indirect or conditional derivation methods.

The logical truths are the theorems of our logical theory. The law of noncontradiction, 6S.7.2 (which is sometimes called, ironically, the law of contradiction), is another logical law.

$$6S.7.2 \quad \sim(\alpha \bullet \sim\alpha)$$

Again, any instance of 6S.7.1 or 6S.7.2 is a tautology of **PL**.

While the laws of the excluded middle and noncontradiction are among the most well known logical truths, there are infinitely many theorems of all of our logical languages: **PL**, **M**, **F**, and **FF**. 6S.7.3 and 6S.7.4 are two more examples of schemas for producing logical truths of propositional logic:

$$6S.7.3 \quad \alpha \supset (\beta \supset \alpha)$$

$$6S.7.4 \quad [\alpha \supset (\beta \supset \gamma)] \supset [(\alpha \supset \beta) \supset (\alpha \supset \gamma)]$$

We began this section by asking about what the laws of logic hold. Now, it seems as if they hold of instances of schemas like 6S.7.1–6S.7.4. But what are these substitution instances? That is, what are we putting in the place of the ‘ $\alpha$ ’s?

## SENTENCES

One option for the instances is that the laws of logic hold of sentences. The word ‘sentence’ is ambiguous between sentence tokens and sentence types. To see the difference, notice that 6S.7.5 contains two sentence tokens, but only one sentence type.

6S.7.5     The cat dances. The cat dances.

The distinction between tokens and types holds of all sorts of objects. ‘Mississippi’ contains four letter types, but eleven letter tokens. The poster of Klimt’s “The Kiss” hanging on a dorm-room wall is a token of the type that was originally instantiated (painted) by Klimt himself. A performance of Mahler’s *Das Lied von der Erde* by your local orchestra is a token of the work.

The laws of logic seem not to hold merely of sentence tokens. Consider disjunctive syllogism, 6S.7.6.

6S.7.6      $\alpha \vee \beta$   
               $\sim\alpha$             /  $\beta$

In order for the conclusion of any instance of DS to follow from its premises, we have to substitute the same thing for  $\alpha$  in both instances, and the same thing for  $\beta$  in both instances. But we can’t substitute the same sentence token. If one token is in the first place, a different token would be in the second place. So, the laws of logic can’t be about sentence tokens.

Perhaps what we want are sentence types. Unfortunately, that seems not quite right either. 6S.7.7, for example, seems to be an instance of disjunctive syllogism.

6S.7.7     The cat either dances or sings.  
              She doesn’t dance.  
              Therefore, she sings.

When we try to see 6S.7.7 as the form 6S.7.6, we notice that there would be different sentence types replacing the  $\alpha$  in 6S.7.6 and different sentence types replacing the  $\beta$ , unless we rewrite the original argument. The first premise isn’t even of the form ‘ $\alpha \vee \beta$ ’, on the surface, since what follows the ‘or’ is not a complete sentence.

We could recast the first premise of 6S.7.7 so that it is more precisely in appropriate form, as the sentence 6S.7.8.

6S.7.8     The cat dances or the cat sings.

Then ‘the cat dances’ replaces  $\alpha$  in the first premise of DS and ‘the cat sings’ replaces  $\beta$ . Unfortunately, the conclusion, ‘she sings’, is still a different sentence type from ‘the

cat sings'. Similar remarks hold for what replaces  $\alpha$ : 'she doesn't dance' has to be rewritten in order to be precisely the negation of 'the cat dances'.

While we can continue to recast 6S.7.7 so that we have exactly the same sentence types in precisely the right positions, we need not do so in order to see that it is a version of disjunctive syllogism. Indeed, we are rewriting 6S.7.7 only because we already see that it is of the form of disjunctive syllogism. It looks like rules of inference and logical laws do not hold of sentence types either.

## PROPOSITIONS

We have seen that logic only implausibly holds of both mental states and of sentences, whether types or tokens. Fortunately, there is a third option. We can take the substituends for our metalinguistic variables, in other words, that of which the laws of logic hold, to be what are ordinarily called propositions. This conclusion should be unsurprising, since we've been calling the variables propositional and calling the logic propositional logic. But the nature of propositions is a controversial subject.

To understand better what a proposition is, notice that 6S.7.9 and 6S.7.10 express the same proposition even though they are different sentences, both types and tokens.

6S.7.9     The cat dances.

6S.7.10    El gato baila.

We can take a proposition to be that which is shared by 6S.7.9 and 6S.7.10. Both sentences say that the cat dances, the first in English and the second in Spanish. And we can say that the cat dances in many different languages. So we can take a proposition to be what gets said by a use of a sentence.

Furthermore, notice that when we talk about what gets said, we naturally use what philosophers call a that-clause. That  $2 + 2 = 4$  may be expressed in numbers or in set theory, or in any natural language. That the sky is blue or that bunnies hop is also expressible in various languages. We can talk about the content of an assertion, the proposition expressed, using a that-clause, though it need not be expressed that way. We can see the use of that-clauses when we try to explain what a sentence means, as in 6S.7.11.

6S.7.11    'snow is white' means-in-English that snow is white

Taking propositions to be meanings of sentences, we can see that a proposition is expressed on the right side of 6S.7.11. Notice that a that-clause is not a complete sentence: that snow is white; that  $2 + 2 = 4$ ; that the door is closed; that I am in Clinton, New York. That-clauses are names of propositions.

Propositions, expressed by that-clauses, can be used as subordinate clauses in a variety of other complex sentences. We can use a proposition in a question, a command, or in an expression of belief or desire; each kind appears in the list at 6S.7.12.

- 6S.7.12 Is it the case that snow is white? (Or 'Is snow white?')  
 Make it the case that the door is closed. (Or, 'Close the door'.)  
 I believe that  $2 + 2 = 4$ .  
 I wish that I were some place warm and sunny.

In some cases, writing complex sentences using that-clauses makes them more awkward. But it can reveal their underlying structure. Sentences such as those in 6S.7.12 seem to contain references to that-clauses as their basic components. Indeed, taking propositions to be expressed by that-clauses allows us to see clearly their role as the objects of belief. We believe that it is raining, or not, or that gold is a precious metal, or that the positive square root of eighty-one is nine. Beliefs and desires are sometimes consequently called propositional attitudes, and taken to be relations between people (believers) and propositions, the objects of our beliefs.

Returning to the use of propositions as meanings of sentences, and to adapt an example of Frege's, consider a man named Gustav telling a friend that he is ill. He might express his condition using the sentence 6S.7.13.

- 6S.7.13 I am ill.

Suppose Gustav asks his friend Lene to call the doctor for him. She might use the sentence 6S.7.14 to explain her call to the doctor.

- 6S.7.14 Gustav is ill.

6S.7.13 and 6S.7.14 are different sentences which express the same proposition, that Gustav is ill. Both sentences mean that Gustav is ill. Propositions, expressible using that-clauses, can be taken to be meanings of sentences which can be expressed in any language.

Still, taking propositions as meanings of sentences is somewhat controversial. Some philosophers reject the claim that there are such things as meanings. We do not see or hear meanings of sentences. We encounter sentence tokens, reading a sentence or hearing it spoken. If the meaning of a sentence is what we grasp, then it seems that we have a grasp of something that is neither a thought nor a physical object.

Frege distinguished three different kinds of objects. The first two are fairly uncontroversial. Concrete objects are those that we can see or otherwise sense. The concrete world contains, as far as language is concerned, both inscriptions and utterances of sentences, sentence tokens.

Second, there are mental objects: ideas or thoughts. We can of course think of mental states or events as concrete objects, tokens of neural firings, say. But a thought seems, at least from the inside, to be a different kind of object from inscriptions or utterances. It is private or subjective.

We have already seen that there are reasons to reject the claim that propositions are concrete objects or mental objects. Like concrete objects, they are objective; we can all believe that snow is white, that Gustav is ill, or that the law of excluded middle is a tautology in bivalent logic. Like mental states, propositions are unavailable to sensory

experience: we can't see them. My thoughts are in my mind; your thoughts are in your mind. Indeed, thoughts are tokens, like inscriptions.

Seeing that propositions are different from both concrete objects and mental states, Frege concluded that they belong to a third realm, of what some philosophers today call abstract objects. Mathematical objects are also naturally taken to be abstract objects. Compare our knowledge of propositions with our knowledge of '2' and ' $2 + 2 = 4$ '. '2' is the name of a number, but is not the number itself, just as I am Russell, but not 'Russell'. We see pizzas and frisbees, but we never see perfect geometric circles. Numbers and geometric objects are thus sometimes called abstract, neither sensible objects nor ideas.

In addition to being mind-independent, propositions seem to be language-independent. They may be expressed by sentences of language, but they technically belong to no language. What a sentence means is not in a language, just as the object to which a term refers is not a linguistic object.

One advantage to taking logic as dealing with propositions is what that allows us to say about ambiguous sentences like 6S.7.15.

#### 6S.7.15 Visiting relatives can be annoying.

6S.7.15 is ambiguous between two propositions; it can be used to express two different meanings. On the first interpretation, we can use it to say that it is annoying when relatives come to visit us. On the second interpretation, we can use it to say that it is annoying to visit our relatives. The ambiguity appears in the sentence token and in the sentence type, but we can distinguish them at the level of propositions. We sometimes say that propositions are thus more fine-grained than sentences: they allow us to make finer distinctions.

If we try to substitute a sentence type in a rule of inference, like disjunctive syllogism, we open ourselves to false inferences because of ambiguity. But not if we're substituting propositions for the schematic variables in our laws of logic. Of course, we write tokens representing, or expressing, those propositions. But the meanings of those sentences, propositions, may be seen as the real subjects of the laws of logic and the rules of inference.

So one standard, if controversial, view about propositions is that they are abstract objects, mind- and language-independent, the meanings of sentences, objects of belief and other propositional attitudes, and expressed using that-clauses. These are natural conclusions, and might be uncontroversial if they did not lead to the conclusion that propositions exist in a third realm inaccessible to sense experience.

Still, mathematical objects seem to be similarly inaccessible to sense experience, so perhaps the view is not completely implausible. While we cannot account for our knowledge of propositions, on this abstract conception, simply by describing sense experiences of them, we cannot account for our knowledge of mathematics that way, either. Philosophers who believe that we have knowledge of the abstract objects of mathematics argue that we can learn about mathematics in part by using names of

mathematical objects, and diagrams, which we can see. Similarly, we might argue that we learn of propositions from our interactions with sentence tokens. The sentence tokens we use in our proofs are names of propositions. Frege puts the point perhaps more poetically: “The thought, in itself immaterial, clothes itself in the material garment of a sentence and thereby becomes comprehensible to us” (“The Thought,” 292).

## Summary

The view that the laws of logic hold of abstract objects called propositions might be called the traditional view, or the standard view. Indeed, the name of our logical language **PL** expresses the entrenchedness of the view described in this section.

The standard view, though, has come under sustained attack for the last century or so. Quine, an innovative and accomplished logician and set theorist, claims that the arguments for the existence of propositions are unsound; there are no such things as propositions. In fact, he calls intensions like propositions creatures of darkness, arguing that belief in meanings in general is a myth. Other philosophers, like Wittgenstein, are also skeptical about meanings.

Most of the worries about meanings are rooted in a problem of access. If propositions are abstract and unavailable to our sensory organs, they seem unknowable. How can we learn of objects in a third realm? The very idea of a third realm independent of the physical world seems preposterous to many people, generating skepticism about propositions, and mathematical objects too.

The propositional skeptics are divided about how to understand logic and its laws. Some, like Quine, find other ways to think about logic, focusing on sentences. Others, like the later Wittgenstein, urge us to rethink the status of logical principles.

On the more traditional side, Stephen Schiffer urges what he calls a pleonastic understanding of propositions, on which they are created by us to be independent of us; we explain our access to them by our ability to create them. Robert Stalnaker has an influential conception of propositions on which they are sets of possible worlds, worlds in which the proposition is true. Such views about propositions lead one to lively contemporary debates in language, epistemology, and metaphysics.

## For Further Research and Writing

1. What are abstract objects? How do they differ from both concrete objects and ideas? Consider mathematical objects as well as propositions. You might also think about artworks including paintings and musical compositions. Consider an objection to the claim that there are abstract objects.
2. How can one understand propositional logic without abstract propositions? Think about both the object language and the metalanguage, and the role of schematic variables. The first chapter of Quine’s *Philosophy of Logic* should be especially useful.

3. How does Frege argue for the existence of a third realm? How does he distinguish propositions from both mental objects and concrete objects? See Frege's "The Thought."
4. How does Wittgenstein's view of meaning, in the beginning of *Philosophical Investigations*, provide an alternative to taking the meaning of sentences to be propositions?

### Suggested Readings

- Bealer, George. "Universals." *Journal of Philosophy* 90 (1993): 5–32. See also "Propositions," *Mind* 107, no. 425 (1998): 1–32. These challenging contemporary defenses of a traditional view of propositions involve modal logic and other contemporary logical tools, and make connections to other kinds of intensions.
- Floyd, Juliet. "Wittgenstein on Philosophy of Logic and Mathematics." In *The Oxford Handbook of Philosophy of Mathematics and Logic*, edited by Stewart Shapiro, 75–128. Oxford: Oxford University Press, 2007. Floyd's essay is a comprehensive survey of Wittgenstein's views on logic and mathematics.
- Frege, Gottlob. "The Thought: A Logical Inquiry." *Mind* 65 (1956): 289–311. Frege's classic defense of the third realm is engaging, if sometimes challenging to follow.
- Grandy, Richard. "What Do 'Q' and 'R' Stand for, Anyway?" In *A Philosophical Companion to First-Order Logic*, edited by R. I. G. Hughes, 50–61. Indianapolis: Hackett, 1993. Grandy defends the claim that logic is about sentences.
- Grayling, A. C. *An Introduction to Philosophical Logic*. Malden, MA: Blackwell, 1997. Chapter 2 discusses propositions and sentence meanings.
- Hanks, Peter. "Recent Work on Propositions." *Philosophy Compass* 4 (2009): 469–86. A review of contemporary work on propositions since the mid-1990s.
- Katz, Jerrold. *The Metaphysics of Meaning*. Cambridge, MA: MIT Press, 1990. Katz defends a mostly traditional view about propositions, responding to skepticism from Wittgenstein and Quine.
- Quine, W. V. *Method of Logic*, 4th ed. Cambridge, MA: Harvard University Press, 1982. In the introduction, Quine defends the view that utterances are the bearers of truth values.
- Quine, W. V. *Philosophy of Logic*, 2nd ed. Cambridge, MA: Harvard University Press, 1986. Quine's classic reflection on the nature of logic starts with a chapter, "Meaning and Truth," in which he discusses alternative understandings of propositions.
- Quine, W. V. "Quantifiers and the Propositional Attitudes." *Journal of Philosophy* 53 (1956): 177–187. Propositions are often taken to be the content of our beliefs and other so-called propositional attitudes. Quine argues that we should avoid commitments to propositions in such cases and proposes alternatives.
- Read, Stephen. *Thinking About Logic*. Oxford: Oxford University Press, 1995. The first chapter ("Truth, Pure and Simple: Language and the World") of Read's accessible introduction to the philosophy of logic discusses various views about propositions.



- Schiffer, Stephen. "Language-Created Language-Independent Entities." *Philosophical Topics* 24 (1996): 149–167. Schiffer's pleonastic attempt to maintain some traditional views about propositions while acknowledging the difficulties of accessing a Fregean third realm.
- Schiffer, Stephen. *Remnants of Meaning*. Cambridge, MA: MIT Press, 1989. Schiffer's book is an excellent introduction to some central problems in the philosophy of language, and begins with some intuitive views about the nature of propositions.
- Stalnaker, Robert. *Context and Content: Essays on Intentionality in Speech and Thought*. Oxford: Oxford University Press, 1999. An excellent collection of challenging essays exploring Stalnaker's influential work on propositions and possible worlds.
- Wittgenstein, Ludwig. *Philosophical Investigations*, 3rd ed. Translated by G. E. M. Anscombe. New York: Macmillan, 1958. The first sections of this broad, engaging, and difficult work explore a roughly behaviorist alternative to thinking of the meanings of sentences as abstract propositions.
- Wolfram, Sybil. *Philosophical Logic: An Introduction*. London: Routledge, 1989. Section 2.4 contains a friendly discussion of propositions (and statements), balancing arguments for and against.