Supplementary Section 7S.10

Quantification and Ontological Commitment

First-order languages contain two types of quantifiers, existential and universal, though the quantifiers are each definable in terms of the other, as we saw in section 4.5. The existential quantifier is of particular interest to philosophers because of its connection to the fundamental question about what exists. We sometimes call the collection of objects that we believe to exist our ontology, or our ontological commitments. The existential quantifier, then, is useful in helping us to identify our ontological commitments.

In this section, I will discuss ontological commitment, the way that language connects to the world, and the relation between these topics and the quantifiers.

METAPHYSICS AND EPISTEMOLOGY

7S.10.1 and 7S.10.2 are questions that have occupied philosophers for a long time.

7S.10.1 What exists?
7S.10.2 How do we know?

7S.10.1 starts us on the road to metaphysics: Are there minds? Are there laws of nature? Is there a God? Some kinds of things more or less obviously exist: trees and houses and people. The existence of other kinds of things is more contentious: numbers, souls, and quarks. Even the question of the existence of some particular things can be debated: Does Tupac Shakur, for example, exist? (He did, of course, but does he?)

7S.10.2 starts us on the road to epistemology. If we believe a claim, for example that there are minds in the world in addition to bodies, then we should have some reasons for believing that claim. Answers to 7S.10.1 are thus tied to answers to 7S.10.2. If I claim that electrons exist, I should be able to demonstrate how I learned about them or how someone discovered or posited them. When we deny the existence of some purported thing, we appeal to a lack of reasons. If you deny my claim that the tooth fairy exists, you might mention that we never see such a thing, or that there are better explanations of the coins under my pillow.

To resolve disputes about what exists, we need to agree on which reasons are good and which are bad. We need a method for determining what exists, rules for the debate over what exists. Our answer to the question whether Tupac exists, for example, depends on what we say about all people who are no longer alive.

Let's follow the Tupac example a little further. Since he is no longer alive, we could naturally think of him as not existing. We can't, for example, see him or talk with him.

There are opposing reasons that might lead us to think that he does exist. For example, we could think of the world as four-dimensional, with three spatial dimensions and one temporal dimension. What exists, on this view, is everything that has any spatial and/or temporal coordinates. In a four-dimensional world, people are extended (temporally) through some portion of the world. It is typical to call such a conception a space-time worm. Whether a person's space-time worm is present at any particular temporal point, like this one right now, is irrelevant to her/his existence. Tupac exists because he has some location in history, unlike the tooth fairy.

We are faced with a question of whether to conceive of the world three-dimensionally, in which case Tupac does not exist, or four-dimensionally, in which case he does. Metaphysics is, in part, the study of our answers to such questions. Epistemology is, in part, the systematic study of the reasons one has for choosing one or the other. It is the study of justification of our beliefs.

GRAMMAR, LOGIC, AND THE ONTOLOGICAL ARGUMENT

Frege, in addition to developing modern logic, contributed centrally to what has become known as the linguistic turn in philosophy. In the twentieth century, many philosophers turned to the study of language in order to engage metaphysical and epistemological questions. If we could become clearer about how language works, some philosophers believed, we could answer some of our long-standing metaphysical and epistemological questions.

One of the early insights made by philosophers of language, perhaps properly ascribed to Bertrand Russell, is that grammatical form is not a sure guide to logical form. Grammatically, for example, 7S.10.3 and 7S.10.4 are parallel.

7S.10.3 Saul Kripke exists.
7S.10.4 Saul Kripke is a philosopher.

Both contain a term for the same subject, Saul Kripke. The first contains a grammatical predicate of existence. The second contains a grammatical predicate of being a philosopher. But, in first-order logic, we regiment the grammatical predicate of 7S.10.4 using a predicate, as at 7S.10.4′, while we regiment the grammatical predicate of 7S.10.3 using an existential quantifier, as at 7S.10.3′. (There are two translations for

each sentence depending on whether we represent 'Saul Kripke' as a predicate, on the left of each, or as a constant, on the right.)

7S.10.3′	(∃x)Sx	or	(∃x)x=s
7S.10.4′	(∀x)(Sx ⊃ Px)	or	Ps

Thus, our typical regimentations of natural languages into first-order logic presuppose that predications of existence are really different from other kinds of predications.

Philosophers have argued for a long time about grammatical predications of existence. Many of those debates have focused on an argument for the existence of God called the ontological argument, which we saw in section 7.4 of *IFLPA*. The ontological argument traces back at least as far as St. Anselm in the eleventh century, and it was central to Descartes's *Meditations* and the works of Spinoza, Leibniz, and many other philosophers. Here, let's start with a version we can attribute to Descartes, 7S.10.5.

7S.10.5 DA1. The concept to which 'God' refers is of a thing with all perfections.
DA2. Existence is a perfection; it is perfect to exist while not-existing would be an imperfection.
DA3. So the concept of God includes existence essentially.
DA4. If a concept includes existence essentially, to deny that the object which that concept represents exists is a contradiction.
DA5. Contradictions are false.
DAC. So, God exists.

In addition to its many defenders, the ontological argument has had many detractors. 7S.10.5 begins with a simple definition and quickly concludes, from an analysis of the logic of our language, that God exists. That conclusion has seemed to many philosophers far too quick. Since David Hume's time, in the eighteenth century, a standard objection to the ontological argument has focused on the difference between existence and other kinds of predications.

> Though certain sensations may at one time be united, we quickly find they admit of a separation, and may be presented apart. And thus, though every impression and idea we remember be considered as existent, the idea of existence is not derived from any particular impression. The idea of existence, then, is the very same with the idea of what we conceive to be existent. To reflect on any thing simply, and to reflect on it as existent, are nothing different from each other. That idea, when conjoined with the idea of any object, makes no addition to it. Whatever we conceive, we conceive to be existent. Any idea we please to form is the idea of a being; and the idea of a being is any idea we please to form. (Hume, *Treatise on Human Nature*, Book I, Part II, section VI)

Hume's allegation here is that our predications of existence are naturally to be taken as unique kinds of predications. They do not augment their subject. They just repeat the idea of the subject. On Hume's view, saying that Saul Kripke exists is to say nothing more than 'Saul Kripke'. To say that God exists is not to ascribe any further property. We cannot, merely by analyzing our language, conclude whether something exists or not.

Immanuel Kant, following Hume and echoing an objection made to Descartes by Pierre Gassendi, argues that existence is not a predicate. It is not a property the way that the perfections are properties. Existence cannot be part of an essence since it is not a property. Whether we think of a thing as existing or not, as necessarily or possibly existing or not, we are thinking of the same thing. "A hundred real thalers do not contain the least coin more than a hundred possible thalers" (Kant, *Critique of Pure Reason* A599/B627). (A thaler was a unit of European currency.)

Kant distinguishes between real (or determining) predicates and logical predicates. A logical predicate is just something that serves as a predicate in grammar. In 7S.10.3, we are predicating (grammatically) existence, but we are not saying anything substantive about Kripke. In 7S.10.4, we use a real predicate.

Any property can be predicated of any object, grammatically. 7S.10.6 is a grammatical sentence, even if it is nonsensical.

7S.10.6 Seventeen loves its mother.

'Loves one's mother' is a real predicate, and 7S.10.6 is a grammatical sentence. But it is grammatical nonsense.

Existence is a grammatical predicate. It works in natural language like a predicate. But for Kant and Gassendi before him, it is not a real predicate. DA2 and DA3, in 7S.10.5, treat existence as a real property, contrary to Hume's claim. When we want to analyze precisely what we believe, we do not take claims of existence as ascribing a real property. So grammatical form is not the same as logical form. One cannot do metaphysics through grammar alone.

Part of Kant's support for his assertion that existence is not a predicate is that existence is too thin. We do not add anything to a concept by claiming that it exists. The real thalers and the possible thalers must be the same thalers in order that the concept be the concept of that object. If there are more thalers in the real thalers, then the concept and the object would not match. We do not add thalers or change the concept when we mention that the thalers actually, and not just possibly, exist.

The claim that existence is not a predicate becomes manifest in contemporary first-order logic. Following Frege, properties like being a god, or a person, or being mortal or vain, are translated as predicates. But existence is taken care of by quantifiers, not predicates. To say that God exists, we might use 7S.10.7 or 7S.10.8.

7S.10.7 $(\exists x)x=g$
7S.10.8 $(\exists x)Gx$

In both cases, the concept of God is represented independently of the claim of existence. In 7S.10.7, we take God as an object in the domain of quantification. In 7S.10.8, we use a predicate that stands for the property of being God. In neither case do we assert the existence of God by the mere use of a name or term. (There are reasons to be wary of names within first-order logic, though; see the appendix to this section for a proof of the existence of God out of first-order logic that depends merely on our typical vocabulary and rules for **F**.)

By taking first-order logic as our canonical language, the one in which we represent our ontological commitments, we vindicate the claim, from Gassendi and Hume and Kant and Russell, that existence is not a real predicate, that grammatical form is distinct from logical form.

WORRIES ABOUT THE KANT-FREGE SOLUTION

In first-order logic, then, it is typical to inoculate constants and predicates from questions about existence. Questions about existence are focused on the quantifiers and the use of a name, like 'Tupac Shakur' or even 'God' is no indication of the existence of the object.

This view leads directly to a worry about names without bearers, sometimes called the problem of negative existentials. If Hume and Kant are correct about names, then the proper analysis of 7S.10.9 is puzzling.

7S.10.9 The Easter Bunny does not exist.

7S.10.9 has a clear and uncontroversial truth values. But how are we to understand the term 'the Easter Bunny'? The most obvious thing for 'the Easter Bunny' to refer to is the Easter Bunny. But there is no such thing. And as Hume claimed, "To reflect on any thing simply, and to reflect on it as existent, are nothing different from each other." But when I reflect on a non-referring singular term, I am not reflecting on it as existent. Indeed, it seems part of our very concept of the Easter Bunny that it is nonexistent.

Kant argued that the possible and real thalers are represented by the same concept. When we ascribe existence, though, we do seem to go beyond the mere stating of a name. In part, that's because a mere name is subsentential, not a full proposition. 'God' and 'God exists' do not say the same thing because 'God', being just a singular term, cannot be used to say anything. Moreover, , as in 7S.10.9, we can deny the existence of something we name.

When my daughter and I discuss the existence of the tooth fairy or the Easter Bunny, we are debating something substantive. If we are going to debate the existence of something, whether it be the Easter Bunny, the tooth fairy, or black holes, we seem to consider an object and wonder whether it has the property of existing. We thus have to consider objects that may or may not exist. Some terms refer to people who are no longer alive. Some terms refer to fictional characters. Others refer to objects we cannot see directly, like quarks. Others refer to objects we cannot perceive in principle, like numbers.

Some philosophers, approaching these questions, distinguish between two different kinds of existence. There is real existence, for you and me and Earth, say. And there is a broader and thinner concept, subsistence, for the referents of any term. Most prominently, Alexius Meinong attributes subsistence to the referent of any meaningful term, dead folks and fictional objects included. He would say that Tupac has the property of subsisting without having the property of existing.

Meinong's work is a point of contention and a starting point for one of the most influential views about the importance of logic for philosophy. W. V. Quine, in his seminal paper "On What There Is," takes first-order logic as canonical, and its existential quantifier as categorical.

THE RIDDLE OF NON-BEING

Quine starts by focusing on the non-referring name 'Pegasus', as in 7S.10.10, parallel to 7S.10.9.

7S.10.10 There is no such thing as Pegasus.

Part of Quine's worry is semantic. How can I state 7S.10.10, or any equivalent, without committing myself to the existence of Pegasus? If we take existence as a predicate, 7S.10.10 seems to say that there is some thing, Pegasus, that lacks the property of existence. But Pegasus is not anything, and it is natural to think that I cannot say something about nothing. In speaking of Pegasus, I am talking about a particular thing. It has to have some sort of existence in order for 7S.10.10 to be sensible.

One option for understanding the term 'Pegasus' is to take it to refer to the idea of Pegasus. John Locke, and many of the modern philosophers who followed him, took words to stand for ideas in our minds. If 'Pegasus' refers to my idea of Pegasus, we can best understand 7S.10.10 as claiming that the idea is not instantiated.

But taking names to refer to the ideas we associate with those names demonstrates a basic confusion of ideas and objects. 'The Empire State Building is tall' refers to an object, not an idea. 'Pegasus is a winged horse' has the same grammatical structure. Why would it refer to an idea, rather than an object? How do we know when a name refers to an external object, like 'The Empire State Building', and when it refers to an idea of an object, like 'Pegasus'? Some singular terms appear to refer to objects even though we do not know if those objects really exist. We do not know whether there is life on other planets, but 'the first planet discovered by people on Earth that supports life' does seem like a sensible term.

More important, against the suggestion that terms like 'Pegasus' refer to my idea of Pegasus, I do have an idea of Pegasus. If 7S.10.10 referred to my idea, then it would, on a natural interpretation, be false; I do have an idea of Pegasus. The problem with 7S.10.10 is that it is true just because there is no object in the world corresponding to my idea.

Meinong's solution to the puzzle, distinguishing between existence and subsistence, is problematic. Quine points out that we also have terms for impossible objects,

like a round square cupola. Since such items can't possibly exist, it's hard to see how they can subsist. We might take terms for impossible objects to be meaningless. But if we take 'round square' to be meaningless, even though 'round' and 'square' are meaningful, we have to abandon the compositionality of meaning, that the meanings of longer strings of our language are built out of the meanings of their component parts. 'Round' is meaningful and 'square' is meaningful, but 'round square', since there can be no such thing, would be meaningless.

Quine says that the abandonment of meaning for such terms is ill-motivated. His main argument consists of his positive account of how to deal with names that lack referents, and how to deal with debates about existence claims generally.

QUINE'S METHOD

One method for determining what we think exists, a method favored by Locke and Hume and Quine's mentor Rudolf Carnap, relies mainly on sense experience. For these philosophers, all claims about what exists must be justified directly by some kind of sense experience. The claim that my knowledge of oranges must derive from my experiences of oranges seems plausible enough. Further, we could use the same claims to defend our beliefs that there is no Loch Ness monster: no one has any direct sense experience of Nessie. But these empiricists had difficulty explaining our knowledge of mathematics and atoms. We do not have any sense experience of the abstract objects of mathematics, and yet we know many facts about them. We have only the merest and most indirect sense experience of atoms.

Another method for determining our beliefs about what exists is favored by Descartes and the great logician Kurt Gödel. These rationalists rely on human reasoning in addition to sense experience. Rationalists have an account of our beliefs about numbers, since they are object of our pure thought. We know of some objects just by thinking about them. But rationalists are often accused of mysticism. Indeed many rationalists, historically, claimed to have certain knowledge of the existence of God. A seemingly magical ability to know something independently of sense experience can be used to try to justify beliefs in ghosts and spirits, as well as numbers and electrons.

In contrast to both rationalists and traditional empiricists, Quine's method for determining ontological commitments invokes the univocality of existential quantification. "To be is to be the value of a variable" (Quine, "On What There Is," 15).

I will attempt to answer two questions about Quine's method. First, what variables are relevant to the question of what exists? Second, what does it mean to be a value of a variable?

The answer to the first question is fairly straightforward. Quine, like his empiricist predecessors, is primarily concerned with the best theories for explaining our sense experience. Our capacities for learning about anything start with sense experience, and what we want, mainly, out of our theories of the world is an ability to explain and predict such experiences. Unlike traditional empiricists, Quine does not pretend to

reduce all claims of existence directly to sense experiences, or insist that only the objects we perceive exist. Instead, he appeals to the most successful and robust systematization of our sense experiences: empirical science. In science, we organize our experiences into the most simple and powerful theories we can develop. Quine argues that our best beliefs about the nature of the universe and the objects in it are found in our best scientific theory. Thus, to know what exists, we look at that best theory and decide what it presupposes, or what it posits. Our best ontology is found in our best theory.

APPLYING QUINE'S METHOD: THE PROBLEM OF EMPTY REFERENCE

While Quine's view that our best beliefs about what exists are to be found in our best science is more than merely plausible, determining what those theories are and what they say exists is not so easy. One problem is that we might encounter many different, competing theories of the world. Determining which theory is best is tricky. We want our theories to be simple and elegant, and yet explanatorily powerful. We want to unify various diverse phenomena. Even among the most powerful and elegant theories, there may be competitors. Quine, indeed, raised questions about whether our best theories are physicalistic or phenomenalist. A physical theory makes claims about the external, material world. A phenomenalist theory makes claims about our experiences of the world. Since we know directly only our own experiences, perhaps our best theory should refer only to our experiences, not to some posited causes of those experiences.

Even given an ability to choose among competing theories, there are questions about how to formulate and read a theory. Quine urges that the least controversial and most effective way of formulating a theory is to put it in the language of first-order logic. He motivates his appeal to first-order logic in part with a discussion of Russell's theory of definite descriptions, the logic of which we saw in section 5.4. In short, here, Russell urges us to see definite descriptions such as 'the king of France' not as names, represented in first-order logic as constant singular terms (or names), but as abbreviations of collections of properties, to be represented in first-order logic as predicates or complex expressions.

To adapt Russell's distinction between grammatical form and logical form for empty names, Quine introduces the predicate 'pegasizes', which stands for a property that holds of all and only things that have the properties that Pegasus does. I used this technique, of regimenting natural-language names as predicates, in 7S.10.8 and on the right sides of 7S.10.3′ and 7S.10.4′. We can regiment 7S.10.10, then, as 7S.10.11, adopting Kant's claim that existence is not a predicate.

7S.10.11 $\sim(\exists x)Px$

7S.10.11 is just the awkward claim 7S.10.10 written in first-order logic. Quine further thinks that we have solved a problem, that we no longer have any temptation to

think that there is a Pegasus in order to claim '~(∃x)Px'. A name can be meaningful, even if it has no bearer.

Famously, Frege distinguished between the meaning of a name, which he called its sense, and the object to which it refers, which we can call its reference. On Frege's view, the meaningfulness of terms with empty reference, like 'Pegasus', is no surprise. But senses are contentious abstract objects. Like Russell before him, Quine prefers to solve the puzzles of empty reference by avoiding senses and distinguishing between grammatical and logical form.

To defend his claim that we can use terms like 'Pegasus' without committing to the existence of something named by 'Pegasus', Quine appeals to his method of determining our commitments by looking at interpretations of first-order logic. In the semantics of first-order logic, as we saw in section 4.7, we call an interpretation on which all of a set of sentences come out true a model of that set. A logically valid formula is one that is true on every interpretation. When Quine says that to be is to be the value of a variable, he means that when we interpret our formal best theory, we need certain objects to model our theories. Only certain kinds of objects will model the theory. The objects that appear in a model of the theory are said, by that theory on that interpretation, to exist.

If, to our first-order logical theory like **F**, we add nonlogical axioms, we create a first-order theory of whatever those axioms concern. If we add mathematical axioms, we can create a first-order mathematical theory. If we add axioms of physics, we can create a first-order physical theory. By couching the axioms of our best scientific theories in first-order logic, we can, theoretically, construct a grand theory of everything. What our best theory says exists will be the objects in the domain of quantification of that theory. And what we should believe exists, our ontological commitments, are the objects in the model of that theory. Such a model will not include Pegasus or the tooth fairy, but it might include dark matter, Tupac, or numbers. To see whether it does, we have to work out our best theory and put it in a proper, perspicuous form.

Unfortunately, the addition of nonlogical axioms strong enough to do the work that scientists require turns our formal system incomplete. We will not be able to prove every truth of the language. But the view of the language of first-order logic as canonical persists among many philosophers. Our best expressions of our ontological commitments will be made in a canonical language, perhaps that of first-order logic. To be is to be the value of a variable of that language.

APPLYING QUINE'S METHOD: OBJECTS AND PROPERTIES

Quine's method for determining the ontological commitments of a theory can be applied to all sorts of questions. Consider again Quine's original worry about Pegasus. While names seem unavoidably referential, Quine urges us to avoid them as the sources of reference. (Again, see the appendix for an excellent reason to avoid names in our canonical language, one consistent with Quine's views.) Instead, we look to the domain of quantification, and the objects that serve as values of our variables. We

regiment our best theory. It will include, or entail, a sentence like 7S.10.10, or its logical equivalent 7S.10.11.

7S.10.11 $(\forall x)\sim Px$

If we want to know whether 7S.10.11 is true, we look inside the domain of quantification. If there is no object with the property of being Pegasus, we call this sentence true in the interpretation. We construct our best theory so that everything in the world is in our domain of quantification and nothing else is.

Universals are among the entities whose existence philosophers debate. Consider, as Quine does, redness. Is redness a thing beyond the particular things, like cardinals, that are red? We can use it grammatically as an object, as in 7S.10.12.

7S.10.12 Redness is prettier than brownness.

A grammatical interpretation of the sensible sentence 7S.10.12 reifies redness, takes it to be an object. A profligate ontologist might thus be led to believe that there are abstract objects in addition to the concrete objects that have their properties. There is appendicitis in addition to people and their appendixes. There is redness in addition to cardinals and fire engines and apples.

Quine insists that just as we can have red cardinals without redness, we can have meaningful statements without meanings. If we again turn to Quine's method, we see a way to neatly express the question. We regiment properties (universals) as predicates. We interpret predicates as sets of objects in the domain. So, the predicate 'is red' is interpreted as the set of all red things. The predicate 'has appendicitis' is taken as the set of all things that have appendicitis. Quine's method demands sets, but not properties. There is a set of red things, but there is no redness.

The difference between sets and properties is that sets are extensional: they are determined exclusively by their members, objects in the domain. If two sets have the same members, they are the same set. In contrast, properties need not be defined extensionally. The set of creatures with hearts and creatures with kidneys is extensionally equivalent, they are the same creatures. But, the property of having a heart is different (intensionally, in terms of meaning) from the property of having a kidney.

Standard interpretations of first-order logic are extensional. We interpret predicates as sets of objects in a domain. Standard interpretations of first-order logic do not reify properties. Still, we can reify them, if we wish, by including them among the objects in the domain of the theory. Thus, first-order logic can maintain a neutrality about existence that makes it compelling as a canonical language for expressing our most considered ontological commitments.

Summary

The introduction of quantifiers into a language that unites propositional and term logics was among Frege's grand achievements. Frege was mainly interested in developing a language for mathematics in support of his logicist claim that all of mathematics is just simple logic in complex form. Ludwig Wittgenstein, in his early *Tractatus*

Logico-Philosophicus (1919), was among the first philosophers to adopt the logic of Frege and Russell for broader philosophical purposes. Quine's work, beginning in the 1930s and lasting through the end of the twentieth century, put logic, especially first-order logic, at the forefront of debates about ontology, about what we should believe exists.

Today, some philosophers dissent from Quine's view of first-order logic as canonical. Many philosophers of mathematics prefer some sort of second-order logic. Other philosophers prefer intensional languages, like those that may be developed using modal operators, and which Quine decried. Still other philosophers deny that formal languages are proper vehicles for such debates. But there's no doubt that such languages, in their precision, can be useful in clarifying ontological claims.

TELL ME MORE

- What are the modal logics that may be used to develop intensional languages? See 6.5: Modal Logics.
- What is the ontological argument in the work of Anselm and Descartes? What are the objections from Gaunilo and Caterus? See 7.4: Logic and the Philosophy of Religion.
- What is Frege's distinction between sense and reference? What is the philosophical importance of Russell's theory of definite descriptions? See 7.6: Names, Definite Descriptions, and Logical Form.
- How are senses controversial objects, like propositions? See 6S.7: The Propositions of Propositional Logic.
- What are Quine's objections to properties and second-order logic? See 6S.13: Second-Order Logic and Set Theory.

For Further Research and Writing

1. What is the ontological status of abstract objects, like numbers or appendicitis? How can we characterize the debate between nominalists and realists? How does Quine's method facilitate the debate? Discuss the role of contextual definition Quine mentions at the end of "Designation and Existence."
2. Are there universals? What is the relationship between the distinction between singular and general statements and the distinction between abstract and concrete terms? Does that relationship help us understand the problem of universals? How does Quine's criterion facilitate the debate? Why does Quine reject meanings in "On What There Is," and how does the rejection of meanings relate to the problem of universals?
3. What is the problem of nonexistence? Consider the solutions provided by McX and Wyman in Quine's "On What There Is." How does Quine's approach differ? How does Quine's approach relate to Russell's theory of definite descriptions?
4. What is a name? What is the relationship between naming and quantification? Discuss Quine's dictum that to be is to be the value of a variable.

Suggested Readings

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Quine, W. V. "Quantifiers and Propositional Attitudes." In Jacquette, *Philosophy of Logic*.

Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus*. Translated by C. K. Ogden. London: Routledge (1922) 1990. The canonical early work of Wittgenstein invokes logic as constitutive of meaningful language and of the world.

APPENDIX

Here is the proof of God's existence out of first-order logic, which I owe to David Rosenthal.

|	1. ~(∃x)x=g	Assumption, for indirect proof, taking 'g' as a name for God
|	2. (∀x)x=x	IDr
|	3. (∀x)~x=g	1, QE
|	4. a=a	2, UI
|	5. ~a=a	3, UI
	6. (∃x) x=a	1–5, IP, DN

QED