THE CHEMIST'S TOOLKIT 6 Work and energy

Work, w, is done when a body is moved against an opposing force. For an infinitesimal displacement through $d\boldsymbol{s}$ (a vector), the work done *on the body* is

$$dw_{\text{body}} = -\boldsymbol{F}\cdot d\boldsymbol{s} \quad \text{Work done on body [definition]} \quad (6.1)$$

where $\boldsymbol{F}\cdot d\boldsymbol{s}$ is the 'scalar product' of the vectors $\boldsymbol{F}$ and $d\boldsymbol{s}$:

$$\boldsymbol{F}\cdot d\boldsymbol{s} = F_x dx + F_y dy + F_z dz \quad \text{Scalar product [definition]} \quad (6.2)$$

The energy lost as work by the system, dw, is the negative of the work done on the body, so

$$dw = \boldsymbol{F}\cdot d\boldsymbol{s} \quad \text{Work done on system [definition]} \quad (6.3)$$

For motion in one dimension, $dw = F_x dx$, with $F_x < 0$ (so $F_x = -|F_x|$) if it opposes the motion. The total work done along a path is the integral of this expression, allowing for the possibility that $\boldsymbol{F}$ changes in direction and magnitude at each point of the path. With force in newtons (N) and distance in metres, the units of work are joules (J), with

$$1\text{ J} = 1\text{ N m} = 1\text{ kg m}^2\text{ s}^{-2}$$

Energy is the capacity to do work. The SI unit of energy is the same as that of work, namely the joule. The rate of supply of energy is called the **power** (P), and is expressed in watts (W):

$$1\text{ W} = 1\text{ J s}^{-1}$$

A particle may possess two kinds of energy, kinetic energy and potential energy. The **kinetic energy**, E_k, of a body is the energy the body possesses as a result of its motion. For a body of mass m travelling at a speed v,

$$E_k = \tfrac{1}{2}mv^2 \quad \text{Kinetic energy [definition]} \quad (6.4)$$

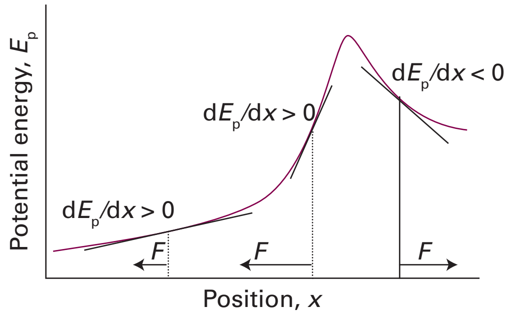
Because $p = mv$ (*The chemist's toolkit* 3), where p is the magnitude of the linear momentum, it follows that

$$E_k = \frac{p^2}{2m} \quad \text{Kinetic energy [definition]} \quad (6.5)$$

The **potential energy**, E_p, (and commonly V, but do not confuse that with the volume!) of a body is the energy it possesses as a result of its position. In the absence of losses, the potential energy of a stationary particle is equal to the work that had to be done on the body to bring it to its current location. Because $dw_{\text{body}} = -F_x dx$, it follows that $dE_p = -F_x dx$ and therefore

$$F_x = -\frac{dE_p}{dx} \quad \text{Potential energy [relation to force]} \quad (6.6)$$

If E_p increases as x increases, then F_x is negative (directed towards negative x, Sketch 6.1). Thus, the steeper the gradient (the more strongly the potential energy depends on position), the greater is the force.



Sketch 6.1

No universal expression for the potential energy can be given because it depends on the type of force the body experiences. For a particle of mass m at an altitude h close to the surface of the Earth, the gravitational potential energy is

$$E_p(h) = E_p(0) + mgh \quad \text{Gravitational potential energy} \quad (6.7)$$

where g is the **acceleration of free fall** (g depends on location, but its 'standard value' is close to 9.81 m s^{-2}). The zero of potential energy is arbitrary. For a particle close to the surface of the Earth, it is common to set $E_p(0) = 0$.

The **Coulomb potential energy** of two electric charges, Q_1 and Q_2, separated by a distance r is

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon r} \quad \text{Coulomb potential energy} \quad (6.8)$$

The quantity ε (epsilon) is the **permittivity**; its value depends upon the nature of the medium between the charges. If the charges are separated by a vacuum, then the constant is known as the **vacuum permittivity**, ε_0 (epsilon zero), or the **electric constant**, which has the value $8.854 \times 10^{-12}\text{ J}^{-1}\text{ C}^2\text{ m}^{-1}$. The permittivity is greater for other media, such as air, water, or oil. It is commonly expressed as a multiple of the vacuum permittivity:

$$\varepsilon = \varepsilon_r \varepsilon_0 \quad \text{Permittivity [definition]} \quad (6.9)$$

with ε_r the dimensionless **relative permittivity** (formerly, the *dielectric constant*).

The **total energy** of a particle is the sum of its kinetic and potential energies:

$$E = E_k + E_p \quad \text{Total energy [definition]} \quad (6.10)$$

Provided no external forces are acting on the body, its total energy is constant. This central statement of physics is known as the **law of the conservation of energy**. Potential and kinetic energy may be freely interchanged, but their sum remains constant in the absence of external influences.