THE CHEMIST'S TOOLKIT 11 Measures of concentration

Let A be the solvent and B the solute. The **molar concentration** (informally: 'molarity'), c_B or [B], is the amount of solute molecules (in moles) divided by the volume, V, of the solution:

$$c_B = \frac{n_B}{V} \qquad (11.1)$$

It is commonly reported in moles per cubic decimetre (mol dm^{-3}) or, equivalently, in moles per litre (mol L^{-1}). It is convenient to define its 'standard' value as $c^\ominus = 1\,\text{mol dm}^{-3}$.

The **molality**, b_B, of a solute is the amount of solute species (in moles) in a solution divided by the total mass of the solvent (in kilograms), m_A:

$$b_B = \frac{n_B}{m_A} \qquad (11.2)$$

Both the molality and mole fraction are independent of temperature; in contrast, the molar concentration is not. It is convenient to define the 'standard' value of the molality as $b^\ominus = 1\,\text{mol kg}^{-1}$.

1. The relation between molality and mole fraction

Consider a solution with one solute and having a total amount n of molecules. If the mole fraction of the solute is x_B, the amount of solute molecules is $n_B = x_B n$. The mole fraction of solvent molecules is $x_A = 1 - x_B$, so the amount of solvent molecules is $n_A = x_A n = (1 - x_B)n$. The mass of solvent, of molar mass M_A, present is $m_A = n_A M_A = (1-x_B)nM_A$. The molality of the solute is therefore

$$b_B = \frac{n_B}{m_A} = \frac{x_B n}{(1-x_B)nM_A} = \frac{x_B}{(1-x_B)M_A} \qquad (11.3a)$$

The inverse of this relation, the mole fraction in terms of the molality, is

$$x_B = \frac{b_B M_A}{1 + b_B M_A} \qquad (11.3b)$$

2. The relation between molality and molar concentration

The total mass of a volume V of *solution* (not solvent) of mass density ρ is $m = \rho V$. The amount of solute molecules in this volume is $n_B = c_B V$. The mass of solute present is $m_B = n_B M_B = c_B V M_B$. The mass of solvent present is therefore $m_A = m - m_B = \rho V - c_B V M_B = (\rho - c_B M_B)V$. The molality is therefore

$$b_B = \frac{n_B}{m_A} = \frac{c_B V}{(\rho - c_B M_B)V} = \frac{c_B}{\rho - c_B M_B} \qquad (11.4a)$$

The inverse of this relation, the molar concentration in terms of the molality, is

$$c_B = \frac{b_B \rho}{1 + b_B M_B} \qquad (11.4b)$$

3. The relation between molar concentration and mole fraction

By inserting the expression for b_B in terms of x_B into the expression for c_B, the molar concentration of B in terms of its mole fraction is

$$c_B = \frac{x_B \rho}{x_A M_A + x_B M_B} \qquad (11.5)$$

with $x_A = 1 - x_B$. For a dilute solution in the sense that $x_B M_B << x_A M_A$,

$$c_B \approx \left(\frac{\rho}{x_A M_A}\right) x_B \qquad (11.6)$$

If, moreover, $x_B << 1$, so $x_A \approx 1$, then

$$c_B \approx \left(\frac{\rho}{M_A}\right) x_B \qquad (11.7)$$