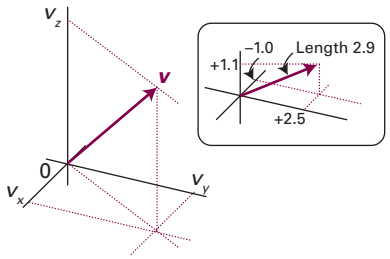
THE CHEMIST'S TOOLKIT 17 Vectors

A vector is a quantity with both magnitude and direction. The vector $\boldsymbol{v}$ shown in Sketch 17.1 has components on the x, y, and z axes with values v_x, v_y, and v_z, respectively, which may be positive or negative. For example, if $v_x = -1.0$, the x-component of the vector $\boldsymbol{v}$ has a magnitude of 1.0 and points in the $-x$ direction. The magnitude of a vector is denoted v or $|\boldsymbol{v}|$ and is given by

$$v = (v_x^2 + v_y^2 + v_z^2)^{1/2} \qquad (17.1)$$

Thus, a vector with components $v_x = -1.0$, $v_y = +2.5$, and $v_z = +1.1$ has magnitude 2.9 and would be represented by an arrow of length 2.9 units and the appropriate orientation (as in the inset in the Sketch). Velocity and momentum are vectors; the magnitude of a velocity vector is called the speed. Force, too, is a vector. Electric and magnetic fields are two more examples of vectors.



Sketch 17.1

Further information

If the polar coordinates of the vector $\boldsymbol{v}$ are θ and ϕ (the colatitude and azimuth, respectively), then

$$\begin{aligned} v_x &= v \sin\theta \cos\phi \\ v_y &= v \sin\theta \sin\phi \\ v_z &= v \cos\theta \end{aligned} \qquad \text{Orientation} \qquad (17.2)$$

and therefore that

$$\theta = \arccos(v_z/v) \qquad \phi = \arctan(v_y/v_x) \qquad (17.3)$$

Brief illustration 17.1: Vector orientation

The vector $\boldsymbol{v} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ has magnitude

$$v = \{2^2 + 3^2 + (-1)^2\}^{1/2} = 14^{1/2} = 3.74$$

Its direction is given by

$$\theta = \arccos(-1/14^{1/2}) = 105.5° \qquad \phi = \arctan(3/2) = 56.3°$$

The operations involving vectors (addition, multiplication, etc.) needed for the text are described in *The chemist's toolkit* 22.