Algebra VI

Multiplying brackets and factorizing

7

Answers to additional problems

7.1 The concentration term [B] occurs twice, so we combine them, writing,

$$-k_{-1}\,[B] - k_2\,[B] = (-k_{-1} - k_2)\,[B]$$

Since both terms within the bracket are negative, we can additionally factorize out the minus sign, and rearrange the order to write, $-[B]\,(k_{-1} + k_2)$

The rate equation therefore becomes,

$$\text{rate} = k_1[A] - [B]\,(k_{-1} + k_2) + k_{-2}\,[C]$$

7.2 Before we factorize this expression, it's useful to recognize how $(\sigma/r)^{12}$ is the same as $((\sigma/r)^6)^2$. Accordingly, we can write the equation as,

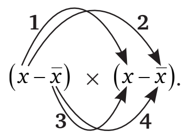
$$\text{energy} = 4\varepsilon\left\{\left(\frac{\sigma}{r}\right)^6 \times \left(\frac{\sigma}{r}\right)^6 - \left(\frac{\sigma}{r}\right)^6\right\}$$

The $\sigma \div r$ terms are repeated, so we can factorize saying,

$$\text{energy} = 4\varepsilon\left(\frac{\sigma}{r}\right)^6\left\{\left(\frac{\sigma}{r}\right)^6 - 1\right\}$$

7.3 The left-hand side merely becomes $(\text{force})^2$ and the '$-k$' term becomes $+k^2$.

The square of the bracket is



1	$= x^2$	**2**	$= -x\bar{x}$
3	$= -\bar{x}x$	**4**	$= +\bar{x}^2$

Terms 2 and 3 are written in a different order but are otherwise the same. The square of the bracket is, $x^2 - 2x\bar{x} + \bar{x}^2$

The overall square of the equation is, $(\text{force})^2 = k^2\,(x^2 - 2x\bar{x} + \bar{x}^2)$.

7.4 To factorize this expression, we first note how 34 000 is exactly 20 times 1 700. We rewrite the expression, as $\Delta G^{\ominus} = (20 \times 1\,700) - 1700\,T$

This latter version of the expression readily factorizes, as $\Delta G^{\ominus} = 1\,700\,(20 - T)$.

7.5 Multiplying the bracket $c(p-1)$ yields $cp - c$. The expression is therefore,

$$DF = cp + 2 - p - cp + c$$

The two cp terms cancel to give, $DF = 2 - p + c$.

- The final '$+$' arises because the $cp - c$ term is subtracted and two minuses make a plus.
- This is the standard form of the **Gibbs phase rule**.

7.6 Equation (7.3) says $(a^2 - b^2)$ as $(a—b)\ (a + b)$. Comparing the bracketed portion of the Rydberg equation with this template allows us to factorize our equation as the difference of two squares,

We say $\frac{1}{n_1^2}$ is a^2 and $\frac{1}{n_2^2}$ in eqn. (7.3) is b^2 so the bracket factorizes as, $\tilde{\nu} = \Re_H\left(\frac{1}{n_1} - \frac{1}{n_2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$

7.7 Within the scheme we have adopted, $a = 4$, $b = -3$, and $c = -2$.

The formula is, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Inserting values for a, b, and c yields,

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 4(-2)}}{2 \times 4}$$

$$x = \frac{-3 \pm \sqrt{9 - (-32)}}{8} = \frac{-3 \pm \sqrt{41}}{8} = \frac{-3 \pm 6.40}{8}$$

so $\quad \frac{-3 - 6.40}{8}$ or $\frac{-3 + 6.40}{8}$

-0.425 or 1.175

and $x = -0.425$ or 1.175.

7.8 We can state $\text{side}_1 = \sqrt{d}$, $\text{side}_2 = 3$, $\text{side}_{\text{longest}} = 2d$.

Inserting terms into the equation for Pythagoras' theorem ($\text{side}_1^2 + \text{side}_2^2 = \text{side}_{\text{longest}}^2$) yields,

$$(2d)^2 = \left(\sqrt{d}\right)^2 + 3^2$$

$$4d^2 = d + 9$$

Rearranging slightly yields, $0 = 4d^2 - d - 9$.

Factorizing with the formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 4$, $b = -1$, and $c = -9$.

Yields, $d = \frac{-(1) \pm \sqrt{(-1)^2 - 4 \times 4 \times -9}}{2 \times 4} = d = \frac{+1 \pm \sqrt{1 + 144}}{8}$

Therefore, $d = 1.63$ or -1.38. A negative length is ludicrous and has no actual meaning so the length d is 1.63.

7.9 Following Worked Example 7.16, 1 mole of un-ionized acid is converted into 1 mole of H^+ ions and 1 mole of anion,

we say $K = \frac{cx^2}{(1 - x)}$

so, $1.75 \times 10^{-5} = \frac{cx^2}{(1 - x)}$, where $c = 0.1$

Cross-multiplying by $(1 - x)$ gives, $1.75 \times 10^{-5} \times (1 - x) = 0.1 \times x^2$.

And elementary rearranging yields,

$$0 = 0.1x^2 + 1.75 \times 10^{-5}x - 1.75 \times 10^{-5}$$

Inserting terms into the formula yields,

$$x = \frac{-1.75 \times 10^{-5} \pm \sqrt{\left(1.75 \times 10^{-5}\right)^2 - 4 \times 0.1 \times \left(-1.75 \times 10^{-5}\right)}}{2 \times 0.1}$$

Subsequent manipulation yields, $x = 1.32 \times 10^{-2}$ or -1.32×10^{-2}.

- We discard the negative root because it cannot represent reality. A concentration at equilibrium cannot itself be negative.
- The concentration of the proton, $cx = 0.1 \times 1.32 \times 10^{-2} = 1.32 \times 10^{-3}$ mol dm^{-3}.
- The proportion of the acid that dissociates is 1.3 percent so ethanoic acid is a weak acid.

7.10 The coefficient of x is 1 and the second term is 18 so, using Equation 7.5, the perfect square is $(x + 9)^2$. This bracket expands to form, $x^2 + 18x + 81$. The difference between this equation and source equation is 11. We factorize as, $(x + 9)^2 - 11 = 0$.

To determine the value of x that satisfies the equation, we say, $(x + 9)^2 = 11$

$$x + 9 = \pm\sqrt{11}, \text{and } x = \pm\sqrt{11} - 9$$

To 4 s.f., $x = 3.317 - 9 = -5.683$

$$x = -3.317 - 9 = -12.317$$