

## The Behaviour of the Atmosphere

## Learning Goals

After studying this chapter, students should be able to:

- apply the ideal gas law and the concept of hydrostatic balance to the atmosphere (pp. 49-54);
- apply the various forms of the hypsometric equation (pp. 54-57);
- account for variations in atmospheric pressure in both the horizontal and vertical directions (pp. 57-59); and
- describe the features shown on weather maps (pp. 59-64).


## Summary

1. The kinetic theory of matter states that all matter is made up of molecules in constant motion. The molecules in gases move more independently than do the molecules in solids and liquids. There is, therefore a unique relationship between temperature, pressure, and density for gases as expressed by the ideal gas law.
2. Two important properties of air arise from the gas law. First, gases are compressible. This is why density decreases with height in the atmosphere. Second, gases expand much more than do liquids and solids when heated.
3. The atmosphere is normally in hydrostatic balance. The force of gravity pulling downward balances the pressure gradient force pushing upward.
4. The hydrostatic equation shows that the rate of change of pressure with height in a fluid depends on the density of the fluid. Because the density of the atmosphere decreases with height, the rate of change of pressure with height is not constant; it decreases quickly at first, then more slowly. In addition, because cold air is denser than warm air, pressure decreases more quickly with height in cold air than in warm air.
5. The hypsometric equation is derived from the hydrostatic equation. This is done using the gas law to substitute temperature and pressure for density. The hypsometric equation shows that the thickness of an atmospheric layer increases with temperature.
6. There are two types of pressure systems: thermal and dynamic. Thermal pressure systems result from heating and cooling. Lows are warm-cored and highs are cold-cored. This makes these systems shallow. In contrast, dynamic low-pressure systems are cold-cored while dynamic highs are warm-cored. This makes these systems deep.
7. Using weather observations taken simultaneously around the world, weather maps are constructed for forecasting purposes. Surface weather maps show the variation of sea level pressure and the location of fronts. Upper-air charts show the variation in height of a pressure surface.

## Key Terms

Atmospheric sounding Measurement of the change with height of certain atmospheric variables, such as temperature, pressure, humidity, and wind (p. 61).

Cold front A front at which cold air is advancing and replacing warm air (p. 45).
Dynamic pressure systems Deep high- and low- pressure systems that develop as a result of complex air motions (p. 59).

Equation of state An equation that provides the relationship between the temperature, pressure, and volume of a substance. The ideal gas law is an equation of state (p. 49).

Fluid A substance that can flow; liquids and gases are both fluids (p. 45).

Front A narrow zone of transition between air of different properties (p. 45).
Hydrostatic balance The state of a stationary fluid when the vertical forces on it are balanced (p. 45).
Hydrostatics The study of stationary fluids (p. 52).
Hypsometry The science of measuring heights (p. 54).
Ideal gas A gas in which there are no attractive forces between molecules (p. 49).
Ideal gas constant The constant, R , in the ideal gas equation (p. 49).
Ideal gas law A scientific law that provides the relationship between the pressure, temperature, and volume (or density) of a gas (p. 45).

Isobars Lines of constant pressure (p. 45).
Kinetic theory of matter A scientific theory that states that matter is composed of molecules and that these molecules are in constant motion (p. 45).

Polar front jet stream A narrow band of very fast westerly wind that occurs in the mid-latitudes in the upper portion of the troposphere (p. 62).

Pressure gradient force A force that occurs due to differences in pressure. The magnitude of this force is proportional to the pressure gradient, and its direction is from high pressure to low pressure (p. 52).

Pressure surface An imaginary surface in the atmosphere upon which the pressure is the same everywhere (p. 54).

Radiosonde A package of instruments that measure pressure, temperature, and moisture and send this information back to the surface through radio transmissions (p. 61).

Synoptic weather map A weather map that gives a visual synopsis of the weather conditions that are occurring at a given time (p. 60).

Temperature A measure of the average kinetic energy of the molecules in a substance (p. 46).
Thermal pressure systems Shallow areas of high or low pressure that are created by cooling or warming, respectively (p. 58).

Thickness The difference in height between two pressure surfaces in the atmosphere (p. 55).
Virtual temperature The temperature used in the ideal gas law to account for the fact that moist air is less dense than dry air (p. 50).

Warm front A front at which warm air is advancing and replacing cold air (p. 45).

Weather station symbols Symbols plotted on a weather map to provide information about observed weather elements (p. 62).

## Key Equations

Ideal gas law

$$
\frac{\mathrm{P}}{\mathrm{~T} \rho}=\mathrm{R}_{\mathrm{d}}
$$

Hydrostatic equation

$$
\frac{\Delta \mathrm{P}}{\Delta \mathrm{z}}=-\rho \mathrm{g}
$$

Hypsometric equation

$$
\Delta \mathrm{z}=\mathrm{z}_{2}-\mathrm{z}_{1}=\left[\frac{\mathrm{R}_{\mathrm{d}} \overline{\mathrm{~T}_{\mathrm{v}}}}{\mathrm{~g}}\right]\left[\left(\ln \frac{\mathrm{P}_{1}}{\mathrm{P}_{2}}\right)\right]
$$

Reduction to sea level

$$
\mathrm{P}_{0}=\mathrm{Pe}^{\left(\frac{\mathrm{a}}{\mathrm{~T}_{\mathrm{v}}}\right) \mathrm{z}}
$$

Reduction to sea level

$$
P=\frac{\mathrm{P}_{0}}{\mathrm{e}^{\left(\frac{\mathrm{a}}{\overline{\mathrm{~T}_{\mathrm{v}}}}\right) \mathrm{z}}}
$$

## Answers to Selected Review Questions (p. 65)

1. At the molecular level, how do gases differ from solids and liquids? How do these differences influence the behaviour of gases?

In gases, the attraction between molecules is very small. In solids, the molecules are strongly attracted to one another. In liquids, the attraction between molecules is slightly less than it is in solids, and the molecules can move more independently from each other.
The differences in attraction are discussed in terms of density, which is an indirect measure of the spacing of molecules. The molecules in gases have the most freedom to move independently and as such, gases are mostly empty space.
3. What relationship is expressed in Boyle's law? What relationship is expressed in Charles's law? Describe both.

In Boyle's law, the relationship between pressure and volume of a gas is expressed. In Charles's law, the relationship between temperature and volume is expressed.

Boyle's law states that if the temperature of a gas is held constant and its volume is increased, then the pressure it exerts will decrease. Charles's law states that if pressure is held constant, an increase in temperature will cause an increase in volume.
5. How does the kinetic theory explain why gases are far more compressible than liquids?

With much more space between the molecules of a gas than between the molecules of a liquid, gases are much more easily compressed.
7. How is the hypsometric equation different from the hydrostatic equation? Why is the hypsometric equation more applicable to the atmosphere?

The hydrostatic equation gives the rate of change of pressure with height for a small change in height. The hypsometric equation gives the difference in height between two pressure surfaces. The hypsometric equation is more applicable to the atmosphere because it takes into account the effect of temperature on the change in pressure with height.
9. What is the difference between thermal pressure systems and dynamic pressure systems?

Thermal pressure systems are shallow areas of high or low pressure that are created by cooling or warming, respectively. Dynamic pressure systems are deep high or low pressure systems that develop as a result of complex air motions.

## Answers to Selected Problems (p. 66)

1. a) Use the gas law and the standard atmosphere to calculate average sea level pressure.
b) Given that the total mass of the atmosphere is $5.14 \times 10^{18} \mathrm{~kg}$ and that the mean radius of Earth is $6.37 \times 10^{6} \mathrm{~m}$, calculate average sea level pressure.
a) $\mathrm{P}=\rho \mathrm{TR}_{\mathrm{d}}$
$=1.23 \mathrm{~kg} \cdot \mathrm{~m}^{-3}(288 \mathrm{~K}) 287 \mathrm{~J} \cdot \mathrm{~kg}^{-1} \cdot \mathrm{~K}^{-1}$
$=101,666.88 \mathrm{~Pa}$
$=101.7 \mathrm{kPa}$
b) $\mathrm{P}=\frac{\mathrm{mg}}{\mathrm{A}}$

$$
\begin{aligned}
& =\frac{\left(5.14 \times 10^{18} \mathrm{~kg}\right)\left(9.8 \mathrm{~ms}^{-2}\right)}{4 \pi\left(6.37 \times 10^{6} \mathrm{~m}\right)^{2}} \\
& =98.8 \mathrm{kPa}
\end{aligned}
$$

3. What is the density of air in each case below?
a) $P=101.3 \mathrm{kPa}, T=15^{\circ} \mathrm{C}$
b) $P=90 \mathrm{kPa}, T=9^{\circ} \mathrm{C}$

Account for the change in density between a) and b).
a) $\varrho=\frac{101300 \mathrm{~Pa}}{(288 \mathrm{~K})\left(287 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}\right)}$ $=1.2 \mathrm{~kg} / \mathrm{m}^{3}$
b) $\varrho=\frac{90000 \mathrm{~Pa}}{(282 \mathrm{~K})\left(287 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}\right)}$

$$
=1.1 \mathrm{~kg} / \mathrm{m}^{3}
$$

The density in b ) is lower because the pressure in b ) is lower.
5. a) Calculate the decrease in pressure from sea level to 100 m .
b) Calculate the decrease in pressure from 5000 m to 5100 m . Account for your findings.

$$
\frac{\Delta \mathrm{P}}{\Delta \mathrm{z}}=-\rho \mathrm{g}
$$

a) $\Delta \mathrm{P}=\left(1.23 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(100 \mathrm{~m})$

$$
\begin{aligned}
& =1205.4 \mathrm{~Pa} \\
& =1.2 \mathrm{kPa}
\end{aligned}
$$

b) $\Delta \mathrm{P}=\left(0.74 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(100 \mathrm{~m})$

$$
\begin{aligned}
& =725.2 \mathrm{~Pa} \\
& =0.7 \mathrm{kPa}
\end{aligned}
$$

Pressure decreases more quickly near the Earth's surface.
7. Calculate the thickness of the 1000 hPa to 900 hPa layer when the average virtual temperature of the layer is as follows.
a) $0^{\circ} \mathrm{C}$
b) $20^{\circ} \mathrm{C}$

Account for your findings.
$\Delta \mathrm{z}=\left[\frac{\mathrm{R}_{\mathrm{d}} \mathrm{T}_{\mathrm{v}}}{\mathrm{g}}\right]\left[\ln \left(\frac{\mathrm{P}_{1}}{\mathrm{P}_{2}}\right)\right]$
a) $\begin{aligned} \Delta z & =\left[\frac{287 \times 273}{9.8}\right]\left[\ln \left(\frac{1000}{900}\right)\right] \\ & =842.4 \mathrm{~m}\end{aligned}$
b) $\Delta \mathrm{z}=\left[\frac{287 \times 293}{9.8}\right]\left[\ln \left(\frac{1000}{900}\right)\right]$

$$
=904.1 \mathrm{~m}
$$

The thickness is larger in b) because the temperature is warmer in b).
9. Given that sea level pressure is 101.325 kPa and average virtual temperature is $0^{\circ} \mathrm{C}$, what is the pressure at the following heights?
a) 1400 m
b) 1 km
$P=e^{\frac{P_{0}}{\left(\frac{a}{T v}\right)^{2}}}$
a) $\mathrm{P}=\frac{101.325 \mathrm{kPa}}{\mathrm{e}^{\left(\frac{0.0342}{273 \mathrm{~K}}\right)^{1400 \mathrm{~m}}}}$

$$
=85.0 \mathrm{kPa}
$$

b) $\mathrm{P}=\frac{101.325 \mathrm{kPa}}{\mathrm{e}^{\left(\frac{0.0342}{273 \mathrm{~K}}\right)^{1000 \mathrm{~m}}}}$

$$
=89.3 \mathrm{kPa}
$$

## Study Questions

For suggested answers, see below.

1. What is virtual temperature? Why is it used as a variable in equations of state?
2. What are two important properties of air that are evident from the gas law?
3. What are the two forces that keep the atmosphere in hydrostatic balance?
4. What are two important implications of thermal pressure systems on atmospheric circulations?
5. Why is weather forecasting more difficult along Canada's west coast?

## Additional Problems

For answers, see below.

1. What is the density of a parcel of air that has a pressure of 92 kPa and a temperature of $-7^{\circ} \mathrm{C}$ ?
2. The temperature of the air inside a container is $11^{\circ} \mathrm{C}$ and the pressure of the air inside is 102 kPa . What is the pressure of the air in the container if it is warmed to $60^{\circ} \mathrm{C}$ ?
3. What is the pressure of a parcel of air that has a density of $0.53 \mathrm{~kg} / \mathrm{m}^{3}$ and a temperature of $-37^{\circ} \mathrm{C}$ ?
4. The density of water is $1000 \mathrm{~kg} / \mathrm{m}^{3}$. What is the water pressure at a depth of 8 m ?
5. Calculate the virtual temperature if $\mathrm{T}=9^{\circ} \mathrm{C}$ and $\mathrm{r}=6 \mathrm{~g} / \mathrm{kg}$.
6. Calculate the difference in pressure from 3000 m to 3100 m .
7. What is the difference in pressure between the surface and 2000 m ? Assume the density of air at the surface is $1.23 \mathrm{~kg} / \mathrm{m}^{3}$ and that the density of air at 2000 m is $1.01 \mathrm{~kg} / \mathrm{m}^{3}$.
8. What is the thickness of the atmospheric layer from 1000 hPa to 800 hPa if the average virtual temperature of the layer is $-5^{\circ} \mathrm{C}$ ?
9. A weather observing station is located at 1846 m . The pressure at this station is observed to be 798 hPa . If the average virtual temperature of the air in the layer is $3^{\circ} \mathrm{C}$, what is the sea level pressure for this station?
10. If sea level pressure is 100.2 kPa , what is the pressure at an elevation of 1075 m when the average temperature from the surface to 1075 m is $-16^{\circ} \mathrm{C}$ ?

## Answers to Study Questions

1. Virtual temperature is the temperature used in the ideal gas law to account for the fact that moist air is less dense than dry air. It is used to account for the presence of water vapour in the atmosphere. (p. 50)
2. The gas law shows that air is compressible and warm air is less dense than cold air. (p. 51)
3. The two forces that keep the atmosphere in hydrostatic balance the pressure gradient force and the force of gravity. (p. 52)
4. Seabreeze circulation systems develop due to differences in heating between the land and the sea. The planetary-scale circulation develops as a result of the differences in heating between the equator and the poles. (p. 58)
5. It is often hampered by the sparse amount of weather observational data over the Pacific Ocean. (p. 60)

## Answers to Additional Problems

1. $\rho=\frac{\mathrm{P}}{\mathrm{TR}_{\mathrm{d}}}$
$=\frac{92000 \mathrm{~Pa}}{(266 \mathrm{~K})\left(287 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}\right)}$

$$
=1.2 \mathrm{~kg} / \mathrm{m}^{3}
$$

2. $\left(\frac{\mathrm{P}}{\mathrm{T}}\right)_{\text {time1 }}=\left(\frac{\mathrm{P}}{\mathrm{T}}\right)_{\text {time 2 }}$

$$
P_{2}=\frac{(102 \mathrm{kPa})(333 \mathrm{~K})}{284 \mathrm{~K}}
$$

$$
=119.6 \mathrm{kPa}
$$

3. $\mathbf{P}=\rho \mathrm{TR}_{\mathrm{d}}$

$$
=0.53 \mathrm{kgm}^{-3}(236 \mathrm{~K}) 287 \mathrm{Jkg}^{-1} \mathrm{~K}^{-1}
$$

$=35,897.96 \mathrm{~Pa}$
$=35.9 \mathrm{kPa}$
4. $\mathrm{P}=\rho \mathrm{gz}$
$=\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(8 \mathrm{~m})$
$=78,400 \mathrm{~Pa}$
$=78.4 \mathrm{kPa}$
5. $\mathrm{T}_{\mathrm{v}}=\mathrm{T}(1+0.61 \mathrm{r})$

$$
\begin{aligned}
\mathrm{T}_{\mathrm{v}} & =282(1+0.61(0.006)) \\
& =283.0 \mathrm{~K} \\
& =10.0^{\circ} \mathrm{C}
\end{aligned}
$$

6. $\Delta \mathrm{P}=\left(0.91 \mathrm{kgm}^{-3}\right)\left(9.8 \mathrm{~ms}^{-2}\right)(100 \mathrm{~m})$

$$
\begin{aligned}
& =891.8 \mathrm{~Pa} \\
& =0.9 \mathrm{kPa}
\end{aligned}
$$

7. $\Delta \mathrm{P}=\left(1.12 \mathrm{kgm}^{-3}\right)\left(9.8 \mathrm{~ms}^{-2}\right)(2000 \mathrm{~m})$

$$
=21952 \mathrm{~Pa}
$$

$$
=21.9 \mathrm{kPa}
$$

8. $\Delta \mathrm{z}=\left[\frac{\mathrm{R}_{\mathrm{d}} \mathrm{T}_{\mathrm{v}}}{\mathrm{g}}\right]\left[\ln \left(\frac{\mathrm{P}_{1}}{\mathrm{P}_{2}}\right)\right]$

$$
\begin{aligned}
& =\left[\frac{287 \times 268}{9.8}\right]\left[\ln \left(\frac{1000}{800}\right)\right] \\
& =1751.4 \mathrm{~m}
\end{aligned}
$$

9. $\mathrm{P}_{0}=\mathrm{P} \mathrm{e}^{\left(\frac{\mathrm{a}}{\mathrm{T}_{v}}\right) \mathrm{x}}$

$$
\begin{aligned}
& =(79.8 \mathrm{kPa}) \mathrm{e}^{\left(\frac{0.0342}{276 \mathrm{~K}}\right)(1846 \mathrm{~m})} \\
& =100.3 \mathrm{kPa}
\end{aligned}
$$

10. $\mathrm{P}=\frac{100.2 \mathrm{kPa}}{\mathrm{e}^{\left(\frac{0.0342}{257 \mathrm{~K}}\right)^{1075 \mathrm{~m}}}}$

$$
=86.8 \mathrm{kPa}
$$

